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## Introducing the Yield Curve as a Tool in Credit Risk Analysis

- ❑ We generate a yield curve that approximates the risk-free yield curve. Since government bonds are considered virtually default-free, the GOVI benchmark bonds (a selection of the most liquid fixed rate government bonds) form a term structure of nearly risk-free interest rates. Using this term structure, a smooth continuous yield curve is fitted relating an approximated risk-free interest rate to any bond duration term.
- ❑ With the yield curve as a reference, credit/liquidity spreads are calculated for a selection of non-government rated bonds. The width of these spreads compares well to each bond's respective credit rating. Even though this yield curve is an approximation of the risk-free yield curve, it is a simple yet useful tool in the analysis of the market's perception of a bond's default risk.

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# Introducing the RisCura Fixed Income Series

In this new limited bi-monthly report series researched by RisCura and distributed by Legae Securities, we aim to provide a research product that has a long shelf life and will contribute to asset managers' current understanding of valuation and risk management issues. This series covers a selection of fixed income related topics, first of which is an introductory report covering:

- ❑ A definition of the yield curve family
- ❑ A definition of credit spread
- ❑ The construction of an approximate risk free yield curve; and
- ❑ A brief look at existing credit spreads and ratings

The objective of this first document is to comment on widely understood and conventional methods of measuring, viewing and controlling credit risk based returns usually carried out by investors. This will provide the groundwork for in-depth discussions on the following topics covered in future reports.

## Performance Issues

- ❑ Accurately calculate credit and liquidity spreads using the BEASSA zero coupon yield curve which is due to be released in the near future.

This report will contain technical details and examples concerning the bond pricing, risk-free YTM calculations and zero coupon yield curve for non-government bonds. By using the risk-free YTM, an accurate credit/liquidity spread can be calculated reflecting improvements to current methods generally being practised in the industry.

- ❑ Investigate methods to extract default risk components from credit spreads, since these spreads reflect both default and liquidity premia.

To separate bond credit from liquidity spreads both direct and indirect methods will be investigated. Firstly we will analyse the differences of the yields on a variety of direct and parastatal government-backed bonds, which will give us some insight into the impact of liquidity on pricing. Clearly this is not enough and so from this, direct measures to extract the liquidity premium from spreads as calculated (including manipulation of variables like the bid-ask spread or trade-data) will be evaluated. We will also investigate indirect measures such as the bond issue amount or bond price volatility to determine the most appropriate method of separating liquidity and credit spreads for the SA bond market.



## Risk Issues

- Investigate ways of quantifying credit risk in a portfolio, based on our experience with the incremental return achieved by taking on credit risk.

With credit risk issues becoming very topical, the advantages and disadvantages of various models attempting to quantify this will be compared to their specific relevance to the SA bond market. The models' inputs, analysts' assessments and market-based observations will also be critically reviewed. We will also review the impact of derivatives on both pricing and measuring risk.

- Analysis of credit spreads, their volatility and impact on pricing.

The primary source of credit return risk is, rather than default, the impact on pricing of a change in credit spreads through expectation changes. We will try to understand these dynamics a little better, given relatively poor data available in this regard.

- Calculate credit-at-risk values for bond portfolios by using a popular credit risk model.

We will look at a variety of credit models and issues such as default probability, payouts, probability density functions and credit ratings within these models. We will propose a relatively simple framework for measuring and managing credit risk.

## Practical Issues

- Practical comments regarding the management of credit risk in a portfolio.

The Fixed Income Series will end with practical concepts such as yield curve and credit risk management. We will look at a variety of important issues within yield curve, credit and liquidity management. This report will include a discussion on fixed income performance attribution and a separation of yield curve management out-performance from credit risk enhancement.



## Confidentiality Issues and Conflicts of Interest

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- ❑ Any party involved in discussions with RisCura on similar topics has provided sign-off on the reports before release in order to protect their intellectual capital.
- ❑ The reports are based on RisCura's original conceptualisation of current global investment trends as well as our own further development of these ideas. We target research areas that are highly relevant to our business with our goal in the reports being to solve issues in a unique, yet academically sound, manner. We also rely heavily on international academic papers in this pursuit.

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# Introducing the Yield Curve as a Tool in Credit Risk Analysis

## 1 Introduction

In this first report, some time is spent covering the topic of yield curves and how they are used to calculate credit spreads. This introductory report will be followed by reports of a more technical nature, concentrating specifically on addressing credit risk issues in the fixed income environment. This series will play an important role in providing asset managers with additional information regarding fixed income risk related issues.

## 2 An Overview of the Yield Curve Family

When comparing debt instruments with identical properties, except for varying maturity terms, the collection of interest rate versus maturity term is referred to as the term structure of interest rates. A graphical representation of this term structure, showing the interest rate as a function of maturity term, is referred to as a yield curve. Although many different yield curves exist, the three most important ones relating to the SA debt market are discussed below:

### 2.1 The BESA Actuaries Yield Curve

The current Bond Exchange of South Africa (BESA) Actuaries Yield Curve, based upon the All Bond Index (ALBI), is well known and is designed to provide a description of the level of interest rates in the bond market on any particular day [1]. However, the yield curve is comprised of the instruments in the ALBI benchmark and therefore incorporates a mixture of default-risky and riskless liquid bonds into the yield curve. These bonds also have different coupon rates and therefore introduce an additional effect, referred to as coupon bias, into the curve.

At most the yield curve can therefore be used to describe interest rate levels in the bond market, serving as an indicator of marketable yield levels for new bond issues or in the valuation of a portfolio of unmarketable bonds. The Actuaries yield curve cannot however be used in the pricing of instruments that depend on the yield curve.

### 2.2 The Government Yield Curve

Historically, most market participants construct yield curves from their observations of market yields on government bonds. The reasoning is that these bonds are from the same issuer and considered free of default risk. Due to their high tradability, these bonds are also considered very liquid and therefore low on liquidity risk. The Government Index (GOVI) benchmark, which represents the very liquid all-government sector of the All Bond Index (ALBI), is an example of such government bonds [2]. The constituents of the GOVI



**Table 1. The GOVI benchmark as on 31/03/2003 consisted of six fixed coupon rate bonds, all with different maturity dates.**

Bonds	Coupon rate	Maturity
R150	12.0%	28/02/2005
R184	12.5%	21/12/2006
R194	10.0%	28/02/2008
R153	13.0%	31/08/2010
R157	13.5%	15/10/2015
R186	10.5%	21/12/2026

and their corresponding coupon rates and maturity dates on 31/03/2003 are listed in Table 1 below.

All bonds in the GOVI benchmark have different coupon rates, again leading to coupon bias, as was the case with the BESA Actuaries yield curve. The key effect of coupon bias is that due to the varying coupon rates of different bonds in the yield curve, some bonds with similar maturity terms may not trade at the same yield. The government yield curve might therefore not be an exact representation of the risk-free rate for a given maturity term.

This effect of the coupon bias can be partly offset by expressing the government yield curve in terms of duration and not maturity. Duration is a measure of the cash-weighted maturity terms of a bond. There are two types of duration – Macaulay duration and Modified duration. The former is useful in immunisation, where a portfolio of bonds is constructed to fund a known liability. The latter is an extension of the Macaulay duration and is a useful measure of the sensitivity of a bond's price (the present value of its cash flows) to interest rate movements. This notion of expressing the yield curve in terms of duration will be revisited later in the report.

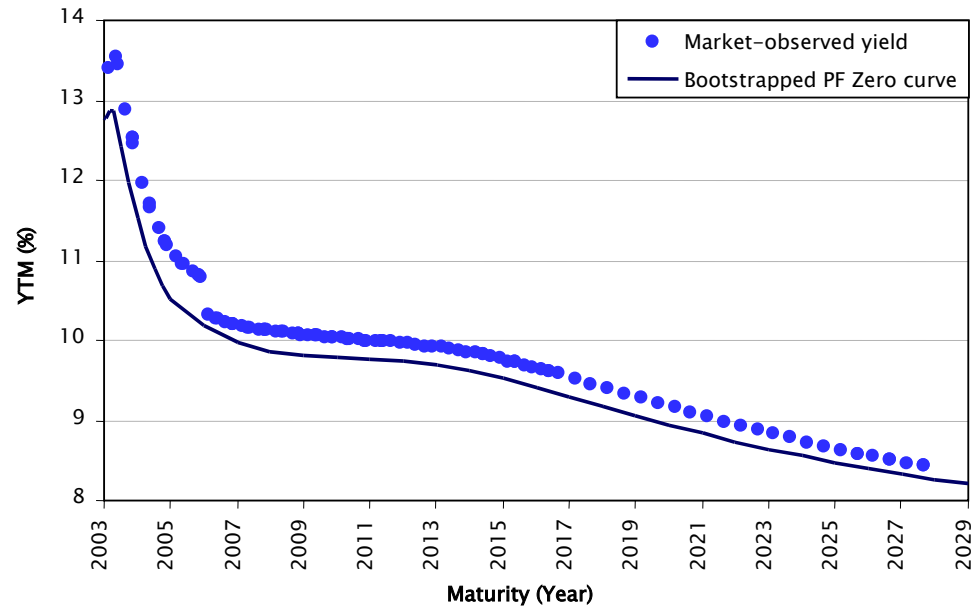
### 2.3 The Zero Coupon Yield Curve

Late in 2001 BESA introduced a standardised zero coupon yield curve for the SA debt market that is derived from the most liquid coupon-bearing government bonds. Consultations regarding the exact specifications of the zero curve have been ongoing, but key issues are close to finalisation and the yield curve will shortly be introduced into the market. In short, the zero coupon yield curve is constructed using the yields from the coupon bearing bonds by an iterative process known as bootstrapping. This determines an appropriate discount rate associated with a unique maturity, which solves the unknown zero rate [3].

In addition to this bootstrapped zero curve, the National Treasury implemented a formal bond stripping program early in 2002 in which the most liquid government bonds' coupon and principal payments are stripped and traded as separate zero coupon bonds on BESA [4]. Since only principal payments occur on these bonds, it is free from coupon bias and due to the credit quality of the issuer, considered to be riskless. In theory the zero coupon



**Figure 1. A comparison between the market-observed zero coupon yield curve and the bootstrapped perfect fit zero curve as on the settlement date of 02/05/2003.**



yield curve can then be observed directly from the market-based yield-to-maturity (YTM) values. However, in practice these bonds are not frequently traded, have fairly wide bid-ask spreads and are therefore not suitable as an indicator for zero rates.

A comparison of the market-observed and bootstrapped zero coupon yield curve is shown in Figure 1. Although not the smoothest zero curve, the "perfect fit" curve that was bootstrapped from a selection of government bonds represents the zero curve that prices its generating instruments exactly. The wide bid-ask spreads between these two curves are evident, indicative of the liquidity issues surrounding the trading of these stripped government bonds.

### 3 Defining the Credit Spread

One area where a yield curve is frequently used is in the calculation of credit spreads for non-government traded bonds. When buyers invest in a non-government bond, the buyer expects to be compensated for the additional default and liquidity risks taken on by demanding a higher yield. The yield difference between this risky bond and an identical but riskless government bond is referred to as credit spread.

Credit spread consists of two separate components, one attributable to default risk and the other to liquidity risk. The ratio of these two components in the credit spread can vary for a number of market and issuer-related reasons. For example, the credit spread for a very default-risky bond, but which has good liquidity, will have large default and small liquidity components. Conversely, the credit spread for a bond that has a small default risk, but which is extremely illiquid will have small default and large liquidity components. Although these examples adequately illustrate the concept of the credit and liquidity



components in the credit spread, the exercise of quantifying these components is more difficult. The proper separation of these components will be the topic of a future paper, but for the time being, the credit spread will be assumed to be wholly attributable to credit risk.

Due to the long investment horizon for bonds as well as possible variations in coupon rates, a non-government bond will rarely, if ever, have an exact matching government bond in terms of its respective cash flow structures. It therefore warrants the creation of a synthetic government bond, which matches the cash flow structure of the non-government bond exactly. The generation of this synthetic government bond can be done in one of two ways.

The more correct method prices the non-government bond using the zero curve, which is bootstrapped from a collection of liquid government bonds. This price then represents the risk-free price for the bond, free of default or liquidity risks. Once the price is determined, the corresponding YTM for the bond is calculated. Again, this YTM then represents the risk-free YTM for the bond. Although this method is the most acceptable method to calculate risk-free rates, it is very calculation intensive. Calculations rely upon the availability of correctly interpolated zero rates corresponding to the bond's cash flow dates. Depending on the compounding method used in the generation of the zero curve, a discount vector can then be generated to discount all cash flows to their present values, considering all the peculiarities regarding the SA bond pricing formula [5]. This price is then used to derive an YTM value using a proper numerical iteration method. The re-pricing method will be revisited in more detail in a future report.

The other, less accurate, method is by approximating the risk-free rate by interpolating the interest rate term structure from a collection of government bonds for the non-government bond's duration. The reason a duration-axis instead of a maturity-axis is used for the yield curve is to counteract the effects that different coupon rates of the government bonds have on the yield curve in general. For bonds with different coupon rates, the YTM and price eventually settled on will impound this coupon rate. In the calculation of the Macaulay duration, each maturity term is discounted by the weight of the cash flow divided by the price of the bond. Although this interpolating method is not as accurate as the re-pricing method, it can still be used as a guide to estimate the credit spread. The remainder of this report will be dedicated to justifying this statement.

#### 4 The Government Term Structure of Interest Rates

As mentioned, the interest rate term structure consists of a collection of identical debt instruments that only differ by maturity term or in this case, duration. For the government term structure, a collection of government bonds called the GOVI benchmark, which is a subset of the ALBI benchmark and consists of the most liquid government bonds, was chosen. As an approximation, the coupon bias between these bonds will be ignored in order to construct a term structure of risk-free interest rates.





#### 4.1 Extending the Government Term Structure of Interest Rates

Referring to the maturity terms of the different government bonds listed in Table 1 above, the R150 bond matures on 28/2/2005. Although this bond has the shortest maturity term in the GOVI benchmark, it is still longer than many of the maturity terms for non-government bonds listed on BESA. Therefore in order to obtain a comprehensive yield curve, especially for maturity terms shorter than that of the R150 maturity term, two additional short-term financial instruments are used for the term structure at shorter durations. These instruments are a risk-adjusted version of the Rand Overnight Deposit rate Index (RODI) and the 91-Treasury bill discount rate. Since these instruments have maturity terms of one business day and 91 days respectively, two extra node points to the left of the R150 node point are introduced, totalling eight instruments for the government term structure.

#### 4.2 The 91-day Treasury Bill

A Treasury bill is a short-term debt obligation of the government and should therefore carry no risk premium or credit spread on the quoted discount rates relative to sovereign debt. The discount rate can be readily converted into a yield rate by using the following expression:

$$y = \frac{d}{1 - \left(\frac{91}{365}\right)d} \quad (1)$$

with  $y$  and  $d$  the yield and discount rate respectively. In Equation (1) the relationship between yield and discount rate can easily be verified using the fundamental definitions of present value,  $PV$ , and future value,  $FV$ , for investments:

$$PV = \frac{FV}{1 + yt} \quad (2)$$

and

$$PV = FV(1 - dt) \quad (3)$$

with  $t$  representing a fraction indicating whether the yield or discount rate is a simple, monthly, quarterly, semi-annual or annual rate. Combining Equations 2 and 3 and solving for the yield for the 91-day Treasury bill leads to Equation (1).

Since the YTM for bonds listed on BESA is quoted on a NACS<sup>1</sup> basis, the 91-day Treasury bill can be converted into a synthetic 3-month maturity term bond by changing the yield  $y$  from NACQ<sup>2</sup> to NACS:

$$\frac{Y}{2} = \left(1 + \frac{y}{4}\right)^2 - 1 \quad (4)$$

<sup>1</sup>Nominal amount compounded semi-annually

<sup>2</sup>Nominal amount compounded quarterly



with  $Y$  the YTM. In Equation (4) the quarterly yield is compounded for two three-month periods assuming that the Treasury bill can be re-invested at the same rate after the first three-month period.

### 4.3 The Rand Overnight Deposit Rate Index

The RODI is the weighted average of the overnight call deposit rates paid by A1-rated local and F1-rated foreign financial institutions where the South African Futures Exchange (SAFEX) places its daily margin deposits received by members [6]. The RODI is quoted on a NACM<sup>3</sup> basis, which can be converted to NACS using the following expression:

$$\frac{Y}{2} = \left(1 + \frac{y}{12}\right)^6 - 1 \quad (5)$$

with  $Y$  the YTM. Since this rate is between non-government entities, it also carries a narrow implicit credit spread and because it is derived from a very liquid instrument, the credit spread is assumed to be mostly due to default risk.

### 4.4 Adjusting the RODI for the Implicit Credit Spread

To account for this implicit credit spread in the RODI, the NACS yield spread between two short-term money market instruments, the 91-day Treasury bill and the 3-month JIBAR rate can be used as proxies for the credit spread between short-term government and non-government debt. By subtracting this yield spread from the RODI, a "risk-free" RODI is obtained that can be used as the first node point in the term structure and yield-curve calculations.

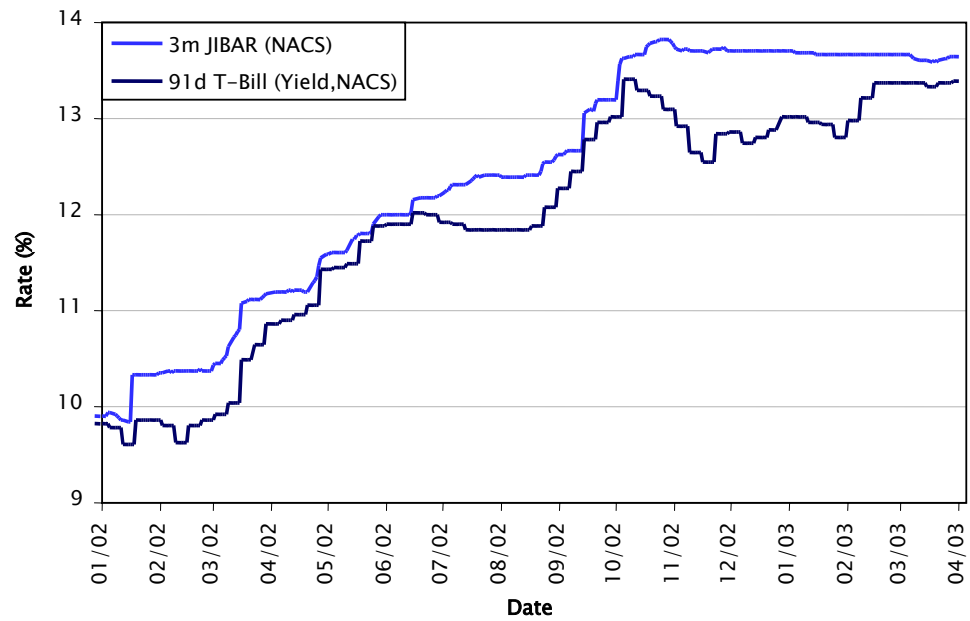
In Figure 2, a comparison between the NACS yields for the 3-month JIBAR and the 91-day Treasury bill for the period 01/01/2002 to 31/03/2003 is shown. As expected the movement of both rates are correlated with the changes in the Reserve Bank's Repo rate during this period. In Figure 3 the yield spread between NACS yields for the 3-month JIBAR and the 91-day Treasury bill, shown in Figure 2, for the period 01/01/2002 to 31/03/2002 is shown. For example, at the end of period on 31/03/2003, the yield spread was 26 bps. In Figure 4 the RODI is shown before and after it was adjusted for the implicit credit spread by subtracting the yield spread between NACS yields of the 3-month JIBAR and 91-day Treasury bill.

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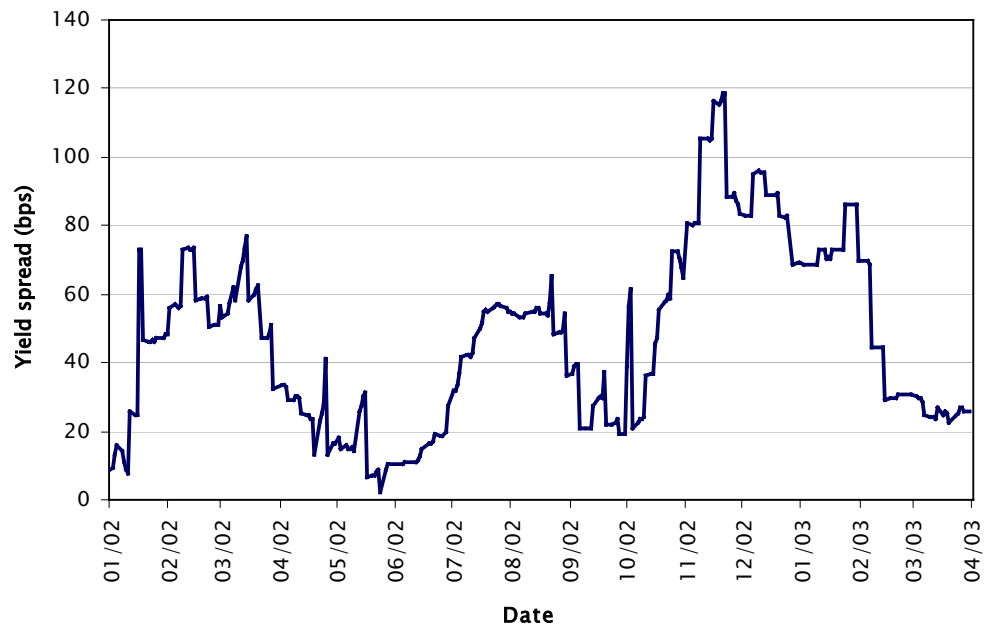
<sup>3</sup>Nominal amount compounded monthly



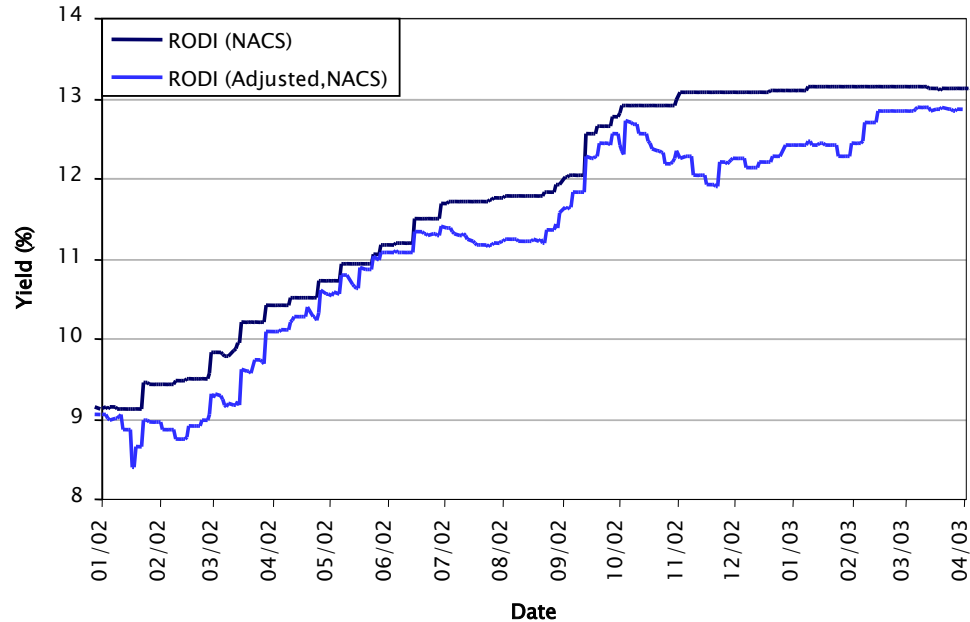
**Figure 2. A comparison between the NACS yields of the 3-month JIBAR and the 91-day Treasury bill for the period 01/01/2002 to 31/03/2003.**



**Figure 3. The yield difference between the NACS yields for the 3-month JIBAR and 91-day Treasury bill for the period 01/01/2002 to 31/03/2003**



**Figure 4. The RODI on a NACS basis before and after it was adjusted for the implicit credit spread by subtracting the yield spread between the NACS yields of the 3-month JIBAR and the 91-day Treasury bill.**



## 5 Generating a Continuous Government Term Structure

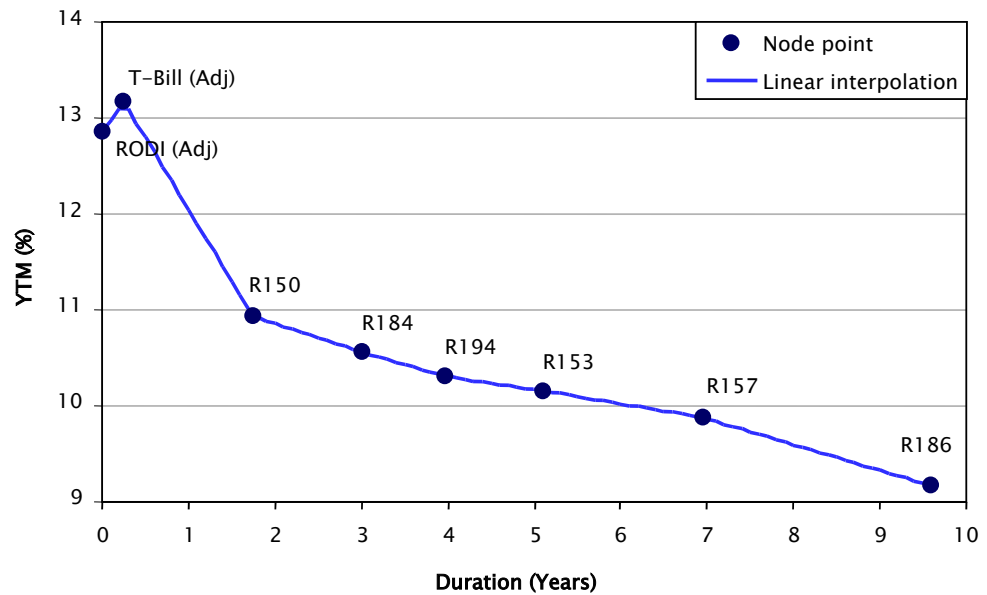
As mentioned, the government term structure only consists of eight instruments. These discrete interest rates therefore need to be interpolated to obtain a continuous government term structure. From here, the "risk-free" rate for any duration can be determined.

### 5.1 Term Structure Interpolation Methods

The simplest method to interpolate these discrete interest rates is to put the YTM for the duration term in question equal to the weighted average of the YTM's for its two nearest neighbours. Mathematically this comes down to linear interpolation, the result of which is shown in Figure 5. The drawback of this method is that the yield curve is not smoothly interpolated and may lead to major interpolation errors, especially at the short-end of the yield curve where large differences between neighbouring instruments occur.



**Figure 5. The government yield curve as on 31/3/2003 constructed from the term structure using linear interpolation.**



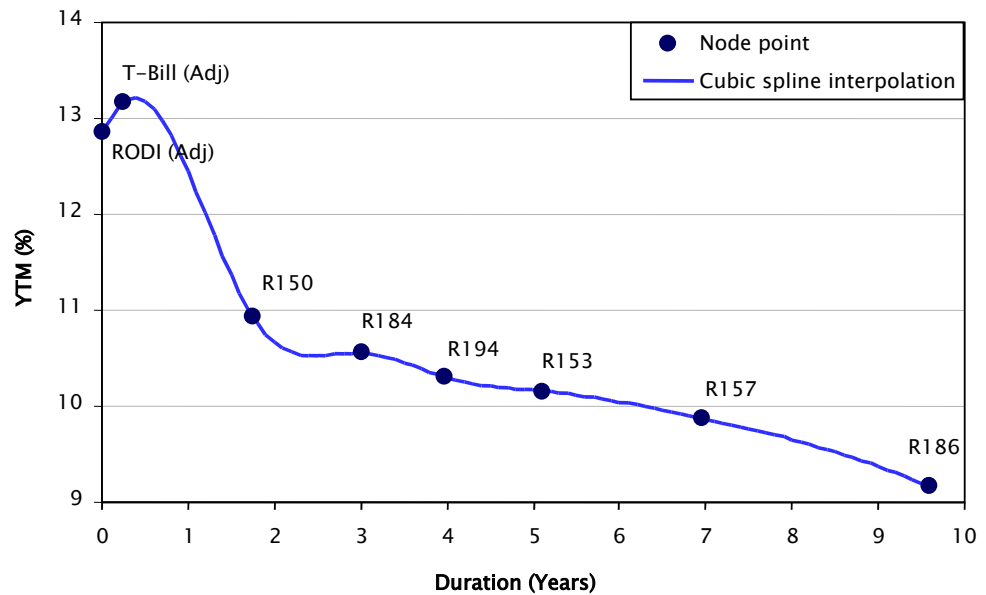
### 5.2 The Cubic Spline Interpolation Method

To fit a smooth curve through the node points in the government term structure, a technique called cubic spline interpolation is used [7]. Supposing that an appropriate number of evenly spaced node points is available, a cubic polynomial can then be fitted through each pair of node points, starting at the first and ending at the last node. When these polynomials are joined together at the node points, they make up a curve called a cubic spline, which represents a smooth curve that intersects the entire given collection of node point [8]. The cubic spline interpolation method is basically the same method applied by BESA [1] in calculating the yield curves published daily. However, since the BESA yield-curve is based upon the ALBI, the benchmark's bonds are clustered along the maturity axis and require a technique called cluster analysis to spread the node points more evenly along it.

By assigning certain constraints to the end-points of the cubic spline, it is possible to alter the curve's shape. The most general cubic spline is the natural cubic spline, which is constructed in such a way that the second derivative at both end points is zero. This results in the ends of the spline being unconstrained so that the free ends are linear. For the yield-curve, it is possible to constrain the fitted cubic spline so that the gradient at the right-hand end is zero. The effect of this constraint on the yield-curve is to flatten the curve at longer maturities. A financial cubic spline denotes a cubic spline that is constrained so that its first derivative at its right-hand end is zero as well as its second derivative at the left-hand end [9]. These additional constraints will slightly change the shape of the curve compared to the natural cubic spline passing through the same set of nodes, so it will not be not be the smoothest curve to interpolate those points. In Figure 6



**Figure 6. The government yield curve as on 31/03/2003 constructed from the term structure using cubic spline interpolation.**



the government yield curve is interpolated using a financial cubic spline. Comparing this version of the yield curve with the previous one shown in Figure 5, it is clear that using the cubic spline interpolation method results in a curve with a high degree of smoothness. However, the cubic spline interpolation also introduces two undesirable artefacts between the nodes for the Treasury bill, R150 and R194 bonds. Although the cubic polynomials between the Treasury bill and the R194 nodes represent the smoothest curves, the YTM predicted by these curves most probably under- and overestimate the market-representative yield.

### 5.3 Comparing the GOVI Yield Curve

As the government yield curve is supposed to approximate the lowest market rate for any maturity term, an obvious test of the applicability of this curve would be to compare it to the YTM's for non-government rated bonds. All non-government issuers rated by Fitch, Moody or Standard & Poors were included in the selection. The result of this comparison is shown in Figure 7, with the bonds colour indexed according to their respective credit ratings. For the mid and long duration end of the curve, the YTM for all the non-government rated bonds are above the yield curve. However at the short end, some bonds actually have YTM's below the yield curve. From an investment perspective this clearly does not make sense as few investors would be willing to accept a yield lower than the risk-free yield, while simultaneously taking on default and/or liquidity risk.



Figure 7. The government yield curve as on 31/03/2003 compared to the YTM's for a selection of non-government bonds. The non-government bonds are grouped according to rating agency information.

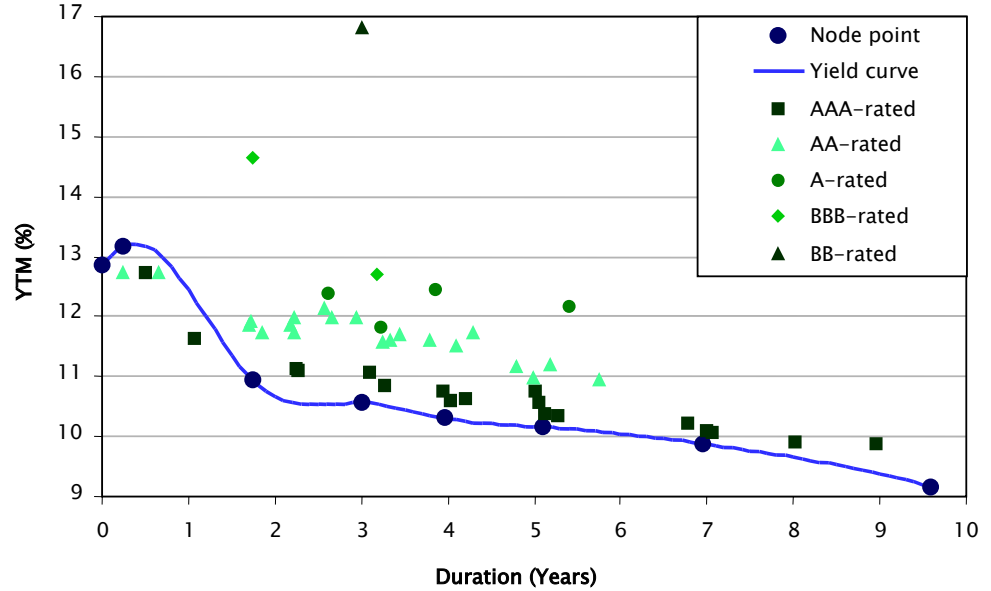
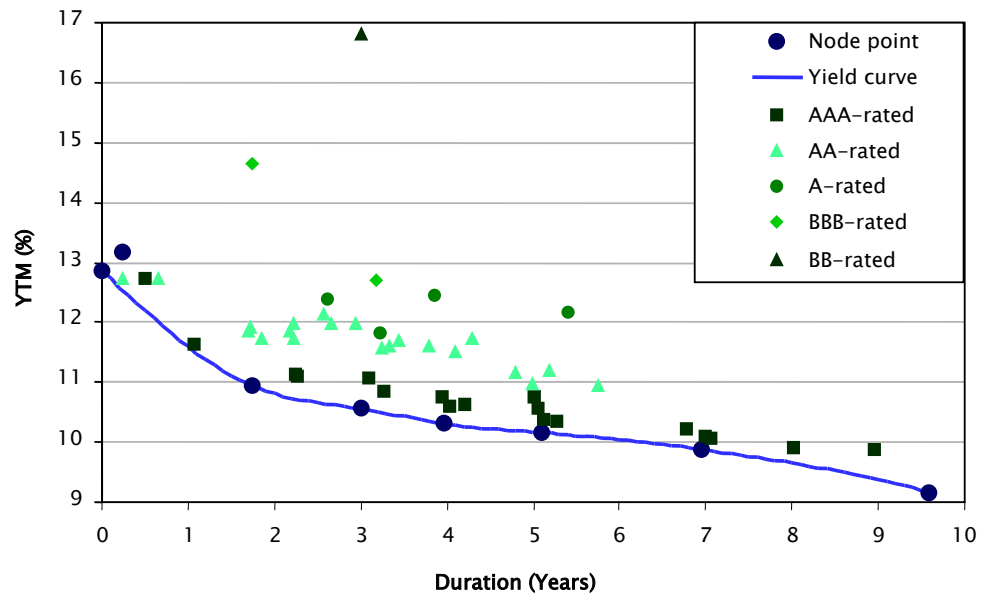


Figure 8. The modified yield curve as on 31/03/2003 compared to the YTM's for a selection of non-government bonds. The yield curve was modified by ignoring the 91-day Treasury bill in the government term structure.



Ignoring the fact that this effect might be attributable to market-related issues, including liquidity, a better description of the yield curve with regards to the YTM's of the non-government rated bonds may be obtained by simply excluding the 91-day Treasury bill yield from the government term structure. Interpolating this modified term structure using the financial cubic spline interpolation method results in the yield curve shown in



Figure 8. Having excluded the second node point from the yield curve, the YTM's of all the non-government rated bonds are now above the yield curve as intended. Furthermore, this also eliminates the artefacts pointed out in Figure 6 regarding the predicted YTM. This modified yield curve will be used as reference in the next section where accurate credit spreads for the non-government rated bonds are calculated.

## 6 Credit Spread Comparison

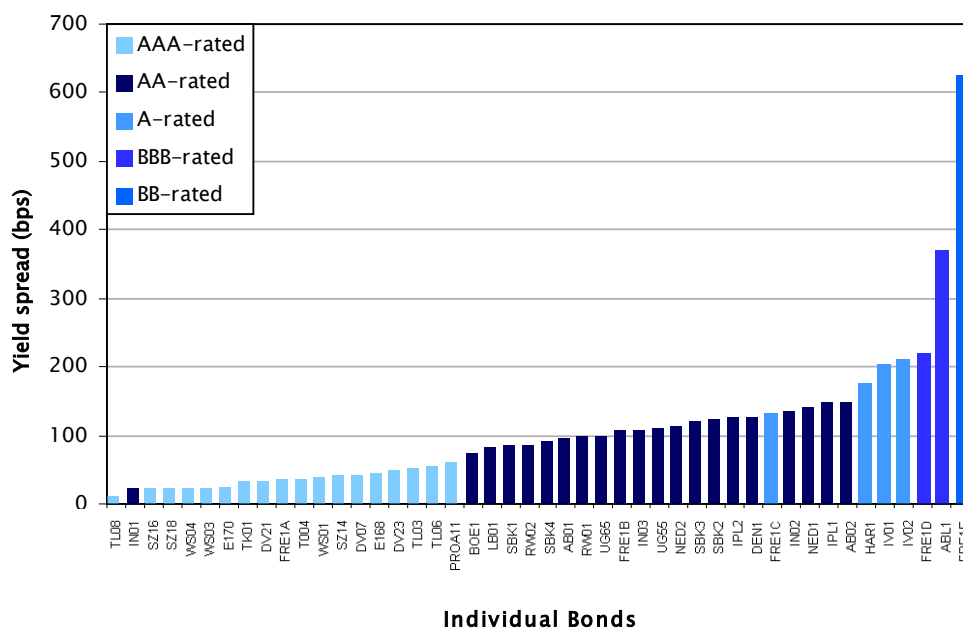
As mentioned, the width of the credit spread is determined by both the default and liquidity components. Since these rated bonds represent the more liquid portion of the non-government bond market, the credit spread should be largely attributable to default risk with the liquidity risk component being subsequently smaller. Clearly this however also depends on the size of the investor as some foreign pension funds and banks may be prejudice of liquidity, even our most liquid instruments. Therefore, in this case, a direct correlation between the width of the credit spread and the bond's corresponding credit rating should exist.

Having determined an approximate continuous risk-free yield curve, the credit spreads between the non-government rated bonds and the modified GOVI yield curve, shown in Figure 8, were calculated and ranked in ascending order based on the width of these spreads. This credit spread distribution is shown in Figure 9 and is colour indexed according to each bond's particular credit rating. The credit spreads for only two bonds, the INCA and FRESCO securitisations IN01 and FRE1C, do not correlate well with their assigned credit rating. Clearly liquidity issues need to be contemplated as well. In the case of the IN01 bond, the problem might also be attributable to the fact that the bond matures on 30/06/2003 and therefore behaves like a short-term money market instrument. The remaining bonds tend to group together according to their respective rating, indicating that a high degree of correlation between the credit spread and credit rating does indeed exist as speculated.





Figure 9. The distribution of credit spreads as calculated using the YTM and yield curves as on 31/03/2003.



## 7 Conclusions

It has been shown that a yield curve giving the approximate risk-free interest rate for any bond duration can be obtained using the GOVI benchmark and the short term RODI rate. Using this yield curve the credit spreads between a selection of non-government rated bonds and the yield curve have been calculated. Assuming that these credit spreads are largely due to the bond's perceived default risk, the credit spreads compared well with the bond's credit rating. This indicates that although the GOVI yield curve is an approximation of the risk-free yield curve, it seems to approximate the expected credit spread well and is a useful tool in quickly analysing the market's perception of a bond's default risk. However, a question remains of an appropriate estimate of credit premium required for any level of measured credit risk.



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