thinkNotes

ideas from RisCura's research team

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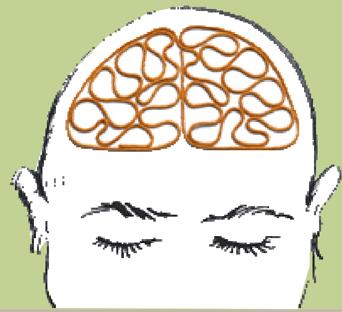

Active Returns: Arithmetic or Geometric?

We are often asked how we go about calculating active returns. Two approaches are generally used, and in this paper we explain both and why we favour the Geometric approach.

The preferred method for calculating active returns[1] (used in performance and performance fee calculations, attribution analysis and mandate specifications) has been the focal point of numerous discussions between RisCura, retirement fund parties and asset managers. Two approaches are generally used when calculating active returns – an arithmetic difference method and a geometric difference method. We prefer the geometric method and in this thinkNote explain why.

As an example of where differences occur when using the two approaches, consider the following two-period investment scenario: in the first period there was no market movement, hence the benchmark had a 0% return, while the asset manager added 1% through skilful investment decisions. In the second period there was a large market movement where the benchmark had a 100% return. However the manager was only able to match the benchmark and added no extra value in the second period. The total compounded return over both periods for the manager was 102%, whilst the benchmark was 100%. If the active return is calculated using the arithmetic method, the result would be 2% whilst if calculated with the geometric method, the result would be 1%.

From this example, we can see that the arithmetic method measures both the manager's skill and the effect of market movements on this skill. The geometric method on the other hand only measures the manager's skill. To understand this difference, we look at the basic definition of the two approaches as well as some of their characteristics.

Arithmetic Approach

In the arithmetic approach, the active return is defined as the profit of the fund over and above the profit of the benchmark, expressed as a percentage of the fund's initial value. Using this definition, the active return includes both the effect of asset manager skill as well as market movements.

[1] Note that 'active return' and 'excess return' are industry terms that are used interchangeably. However, at RisCura we interpret the former to be the difference between the fund and the benchmark, while the latter refers to the difference between fund and a risk-free rate. Therefore the 'excess return' refers to a special case of 'active returns' where the benchmark return is equivalent to the risk-free rate.

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For example: A fund has a return of 7%. So, for an initial market value of R100, its value increases to R107 over the period, giving a profit of R7. At the same time, the benchmark returns 5%, which implies that, if we assume the same initial market value as the fund, its market value would have increased to R105 at the end of the period. The fund's excess profit over the benchmark is therefore is R7 – R5, or R2. Following the definition above, the active return is therefore R2/R100 or 2%.

Mathematically, if R_F^i and R_B^i represents the fund and benchmark return for period i and MV^i is the fund's initial market value, we can use the definition above to express the arithmetic active return R_A^i as follows:

$$R_A{}^i = \frac{\left\{MV^i\left(1+R_F^i\right)-MV^i\right\}-\left\{MV^i\left(1+R_B^i\right)-MV^i\right\}}{MV^i} \tag{1}$$

which easily simplifies to the arithmetic difference between the fund and benchmark returns:

$$R_A{}^i = R_F{}^i - R_B{}^i \tag{2}$$

Geometric Approach

For the geometric approach, the active return is defined in a slightly different way. It is still defined as the profit of the fund over and above the profit of the benchmark, but in this case is expressed as a percentage of the benchmark's final value. Using this definition, the active return includes only the effect of asset manager skill and excludes the effect from market movements. The benchmark's end value is used so that the fund's current profit is assessed relative to the potential profit that would have been obtained had the fund invested in the benchmark.

To illustrate, take the same fund and benchmark of the previous example. Again the excess profit of the fund over the benchmark's profit will amount to R2, but now the geometric active return is this excess profit expressed relative to the benchmark's end market value and not the fund's starting value. Therefore the geometric active return is R2/R105 or 1.9% as opposed to 2% in the arithmetic example.

Similar to the method followed in Equation (1), we can express the geometric active return R_G^i as follows:

$$R_G^i = \frac{\left\{MV^i\left(1+R_A^i\right)-MV^i\right\}-\left\{MV^i\left(1+R_B^i\right)-MV^i\right\}}{MV^i\left(1+R_B^i\right)} \tag{3}$$

As the denominator in the above equation is different to the one in equation (1), it simplifies to the geometric difference (rather than the arithmetic difference) between the fund and benchmark returns:

$$R_G^i = \frac{1+R_F^i}{1+R_B^i} - 1 \tag{4}$$

Also, from equation (2) and equation (4), we can see that the arithmetic active return is related to the geometric active return in the following way:

$$R_A^i = R_G^i \left(1 + R_B^i\right) \qquad (5)$$

Therefore we see that the arithmetic active return will only give results similar to the geometric active return in situations where the benchmark fund's return is close to zero. This can be intuitively understood from the definitions for the arithmetic and geometric methods, because the benchmark's final market value would be close to the fund's initial market value when the benchmark's return is close to zero.

Unique Characteristics

Rather than subjectively listing the advantages and disadvantages of these methods, we'll examine the use of the arithmetic method and compare it to the use of the geometric method.

For single period active returns, the arithmetic approach is easy to calculate, intuitive to understand, and widely used. When the arithmetic active return is calculated over multiple periods, however, care needs to be taken in choosing the method that is used to calculate the active return. This is because multiple period returns take compounding[2] into consideration, by using geometric linking. As a result, the multiple period arithmetic active return calculation is sensitive to the order in which the calculation is made. We therefore need to establish whether the arithmetic active returns were calculated period-by-period and then compounded, or if the fund and benchmark returns were compounded separately before an arithmetic active return was calculated. To illustrate, look at the following example in Table 1 below.

	Return		Active Return			
Period	*Fund*	*Benchmark*	*Arithmetic*		*Geometric*	
1	3%	2%	1%		0.98%	
2	5%	1%	4%		3.96%	
3	3%	6%	-3%		-2.83%	
Compounded	11.39%	9.20%	1.89%	2.19%	2.01%	2.01%

Table 1. Arithmetic and geometric active returns calculated over multiple periods.

The table shows the calculation of the active return between a fictitious fund and benchmark over 3 periods using both the arithmetic and geometric approach. For both approaches, the active return over the 3-month period was calculated in the following order: for the blue cells, the active return was calculated by compounding the monthly active return, and for the green cells, the active return was calculated between the compounded fund and benchmark returns. Notice that although the arithmetic approach leads to ambiguous results when its light and dark grey cells are compared to one another, the geometric approach is consistent and does not depend on the order of calculation.

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[2] Multiple period compound returns are calculated from the single period returns by geometrically linking the single period returns using the following formula:

$$R_T = \left(1 + R_1\right)\left(1 + R_2\right)\left(1 + R_3\right) - 1$$

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Another area where the differences between the arithmetic and geometric methods are highlighted is when active returns are calculated through different market environments. Below, in Table 2, an example is given where the arithmetic and geometric active returns are calculated for bullish and bearish market environments using fictitious fund and benchmark returns.

Market Environment		Bull Market		Bear Market	
Performance		*Out*	*Under*	*Out*	*Under*
Fund Return		3%	1%	-1%	-3%
Benchmark Return		2%	2%	-2%	-2%
Active Return	Arithmetic	1.00%	-1.00%	1.00%	-1.00%
	Geometric	0.98%	-0.98%	1.02%	-1.02%

Table 2. Arithmetic and geometric active returns in varying market conditions.

If we compare the arithmetic active returns across both market environments, we see that the active returns are equal for both outperformance and underperformance. This means that arithmetic active returns are symmetrical in upward trending markets and downward trending markets. On the other hand, when comparing the geometric active returns across both market environments, we see that the active returns are asymmetric, since the magnitude of the active return is less in upward trending markets than in downward trending markets. This difference in the symmetry of arithmetic and geometric active returns across various market environments has some practical implications when fixed and performance-based management fee structures are being considered.

Before we discuss the effect of the active return method on management fees, let's first look at how the management fees work. As the name suggests, a fixed management fee is a flat fee charged by the asset manager, and is a set percentage of the average market value of the funds under management over a period of time. In a bullish market environment, the upward trend will increase the market value of the funds under management, in turn increasing the fixed management fee. Conversely, in a bearish market, the downward force on market prices will decrease the market value, which in turn will decrease the fixed management fee. A performance-based management fee, on the other hand, is a set percentage of the outperformance multiplied by the average market value.

This fee is designed to reward the asset manager when it outperforms some benchmark or performance target, as set out in the mandate between the investor and the asset manager. This reward applies irrespective of whether the outperformance occurs in a bullish or bearish market.

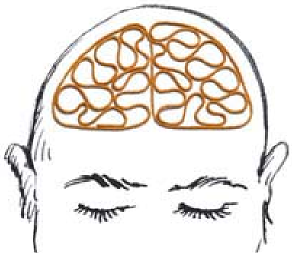
When the arithmetic approach to active returns is used, outperformance in either a bullish or bearish market results in the same performance fee, since the active returns are symmetrical, as seen in Table 2. Combining this symmetrical performance fee with the fund manager's fixed management fee, we see that outperformance in a bullish market actually rewards the manager more than a similar outperformance in a bearish market. This skewness in reward is not as pronounced when the geometric approach is used. Here the manager is rewarded less for outperformance in a bullish market and more for outperformance in a bearish market[3].

[3] The manager is rewarded more for not incurring as high a capital loss as the benchmark in a downmarket, as the geometric method only measures the manager's skill and not the effect of market movements.

When this performance fee is combined with a fixed management fee, which is higher in an upmarket and lower in a downmarket, we have a more symmetrical reward profile for managers across different market environments.

Our Preference

At RisCura we prefer to use the geometric method when comparing the returns of managers because it only measures an asset manager's skill and excludes the effect of market movements. This is because the geometric method measures the return of the manager relative to the return the investor would have received had they been invested in the benchmark. We also believe that the geometric method rewards managers more fairly, on a combination of performance and fixed management fee basis, through all market conditions. Finally, we prefer the unambiguous computation that results when calculating geometric active returns over multiple periods.



There is no problem that considered thought, intelligence, applied knowledge and imagination cannot solve.

Well, that's how we think.

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