

## ANALYSIS OF OVERLAP – AN APPLICATION FOR PORTFOLIO ANALYSIS *by Santosh Ramkissoon*

### WHAT IS OVERLAP?

Overlap is a measure of the commonality between portfolios. Knowing this commonality is important since, one would typically invest across funds with the aim of diversifying. The idea behind diversification is to buy different assets so as to decrease the specific risk associated with each asset. If the overlap between portfolios is too big an investor may incorrectly believe that by holding those portfolios the total risk he/she is exposed to is being diversified away. Clearly there needs to be a measure that can identify the commonality between portfolios.

### CALCULATING OVERLAP

With this in mind, we have developed a technique for calculating the overlap between portfolios. At the onset, it is necessary to point out that analysis of overlap is a fundamental method of determining diversification. In other words, holding portfolios that do not have significant overlap is *necessary* in order to be diversified. However, considering overlap alone might not be *sufficient*. In addition to considering overlap, investors should also consider the degree of correlation between the assets. Nevertheless, for now we restrict our attention to overlap analysis, i.e. the most basic requirement for diversification. The importance of developing a technique for measuring overlap is borne-out by its many applications. Using this technique, the various applications for measuring overlap, as listed below, can be achieved quickly and efficiently.

### APPLYING THE OVERLAP ANALYSIS TECHNIQUE

The process of asset transition involves changing the composition of an existing portfolio to a new allocation. When doing a transition to update an existing portfolio, the overlap analysis technique can be used to quantify the percentage of a portfolio that needs to be bought or sold so as to carry out the transition. Alternatively, an asset transition may involve selling out of a particular fund and investing into another fund. When doing a transition to change from one fund to another, it may be that the fund being invested into charges higher management fees than the one being sold out of. An overlap analysis can be carried out to determine whether there is extensive overlap between the two funds. If an overlap analysis reveals that there is indeed extensive overlap between the funds, transitioning between funds may be deemed unnecessary.

Another application relates to the risk management of investment funds. The overlap analysis technique can be used to test whether the level of overlap between assets or securities in a portfolio are acceptable, i.e. is the portfolio sufficiently diversified? Having a technique that can do this efficiently is crucial in terms of being able to dynamically make changes to the portfolio if the level of overlap is deemed unacceptable. The opposite might apply as well where similar funds are required. The above can also be applied to i.e. Fund-of-Funds where the underlying funds can be viewed as the asset or securities in a traditional fund.



#### Santosh Ramkissoon

*MSc (Physics); currently completing MSc (Financial Mathematics)*

Santosh joined RisCura straight from the University of Cape Town and sits on our research team. Santosh loves to using cutting edge mathematical, statistical and computational techniques to solve challenging problems, and thinks the best thing about RisCura is the "can-do" attitude of its very smart people.

# think Notes.

Overlap A,B,C =  $\frac{58,461.00 + 3 \leftarrow 7,961.50}{7,961.50} = 29,01\%$

3  $\leftarrow$  7,961.50

7,961.50 + 3  $\leftarrow$  7,961.50

Fund A

## PAIR-WISE OVERLAP

Suppose that we have two overlapping funds, i.e. Funds A and B. Figure 1 shows a diagrammatic representation of the overlapping funds. Note that A and B can denote market capitalisation or the holdings within the portfolios. In this article we illustrate the concept for market capitalisation.

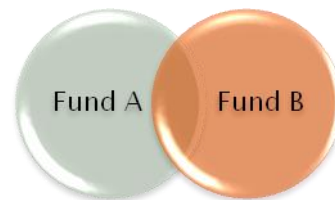


Figure 1: Funds A and B overlapping

The overlap between them is given as the number of elements (holdings or monetary units when using market capitalisation) in common counting both portfolio A and B (i.e. double counting the common elements) divided by the total number of elements in portfolio A and portfolio B.

$$\text{Overlap}_{A,B} = \frac{2 \times (A \cap B)}{(A + B)} \quad 1$$

It is important to test equation 1 for extreme cases, i.e. complete overlap or no overlap. Suppose Funds A and B overlap completely, see figure 2. The numerator of equation 1 is  $2 \times (A \cap B) = A + B$ . Therefore we get cancellation of the numerator and denominator which gives overlap = 100%, as required.



Figure 2: Funds A and B overlap completely

Suppose Funds A and B have no common shares, see figure 3, then the numerator of equation 1 = 0 so that we get overlap = 0%, as required.



Figure 3: Funds A and B have no overlap

Suppose now that these two overlapping funds have holdings as shown in the table below. We can apply equation 1 to calculate the pair-wise overlap between Funds A and B, as has been done in equation 2.

	ACL	AGL	ASA	BIL	IMP	MTN	SOL
Fund A	30	0	0	15	20	60	40
Fund B	0	20	40	80	0	30	10
Overlap	0	0	0	15	0	30	10
Price	R 151.00	R 334.80	R 107.80	R 204.50	R 194.00	R 117.10	R 357.45

$$\text{Overlap}_{A,B} = \frac{2 \times (\text{Market value of shares in common})}{(\text{Market value of fund A}) + (\text{Market value of fund B})} \quad 2$$

Market value of shares in common

# think Notes.

Overlap A,B,C =  $\frac{58,461.00 + 3 \leftrightarrow 7,961.50}{7,961.50} = 29.01\%$

3  $\leftrightarrow$  7,961.50

7,961.50

Fund A

$$= (15 \times \text{BIL}) + (30 \times \text{MTN}) + (10 \times \text{SOL})$$

$$= \text{R } 10,155.00$$

## Market value of Fund A

$$= (30 \times \text{ACL}) + (15 \times \text{BIL}) + (20 \times \text{IMP}) + (60 \times \text{MTN}) + (40 \times \text{SOL})$$

$$= \text{R } 32,801.50$$

## Market value of Fund B

$$= (20 \times \text{AGL}) + (40 \times \text{ASA}) + (80 \times \text{BIL}) + (30 \times \text{MTN}) + (10 \times \text{SOL})$$

$$= \text{R } 34,455.50$$

Substituting the values above back into equation 2 gives:

$$\text{Overlap}_{A,B} = \frac{2 \times 10,155.00}{32,801.50 + 34,455.50} = 30.2\%$$

3

The value of 30.2% implies that nearly a third of the underlying holdings in Funds A and B overlap. Depending on the desired outcome, conclusions can then be drawn and the appropriate action taken based on the calculated figure of 30.2%.

## TOTAL OVERLAP

In the previous section we dealt with overlap between two funds, i.e. pair-wise overlap. As we were only considering two funds, the pair-wise overlap value also represents the total overlap value. The question now is what happens to the total overlap value when we consider more than two funds. Let us consider a scenario with three funds that are potentially overlapping. Consider for example Funds A, B and C as shown in Figure 4.

In the figure below, the *total overlap* is represented by the intersection of all three funds. *Pair-wise overlap* between Funds A and B would be represented by the overlap between Funds A and B alone. Pair-wise overlap for Funds B and C and Funds A and C would work the same way.

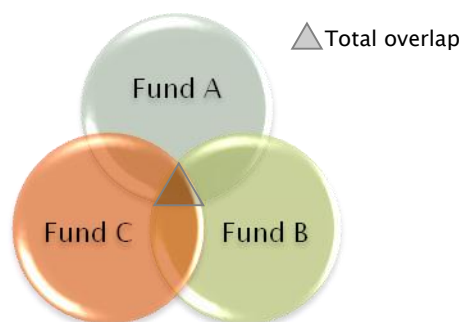


Figure 4: Multi-fund overlap

Using equation 1 the two fund example can easily be extended to include  $N$  funds  $A_1, A_2, \dots, A_N$  in the total overlap calculation.

think Notes.

Overlap A,B,C =  $\frac{58,461.00 + 3 \times 7,961.50}{7961.50}$  = 29.01%

3  $\leftrightarrow$  7,961.50

7,961.50

Fund A

$$\text{Total Overlap}_{A_1, A_2, \dots, A_N} = \frac{N \times \bigcap_{i=1}^N A_i}{\sum_{i=1}^N A_i} \quad 4$$

Similar to the example shown in the previous section, suppose that Fund A and B now have a third overlapping fund, i.e. Fund C, as shown in the table below. We apply equation 4 to calculate the total overlap between these three funds.

$$\text{Total Overlap}_{A,B,C} = \frac{3 \times (\text{Market value of shares in common to all funds})}{(\text{Sum of market value of all funds})} \quad 5$$

	ACL	AGL	ASA	BIL	IMP	MTN	SOL
Fund A	30	0	0	15	20	60	40
Fund B	0	20	40	80	0	30	10
Fund C	0	0	30	10	0	20	20
Overlap	0	0	0	10	0	20	10

**Market value of shares in common to all funds**

$$= (10 \times \text{BIL}) + (20 \times \text{MTN}) + (10 \times \text{SOL})$$

$$= R 7,961.50$$

**Sum of market value of all funds**

$$= \text{Market value of Fund A} + \text{market value of Fund B} + \text{market value of Fund C}$$

$$= R 82,027.00$$

Substituting the values above back into equation 5 gives:

$$\text{Total Overlap}_{A,B,C} = \frac{3 \times 7,961.50}{82,027.00} = 29.1\% \quad 6$$

The value of 29.1% is lower than the pair-wise overlap value between Funds A and B, which was 30.2%. In fact, the total overlap value will always be at least lower than or at most equal to the smallest overlap value of any of the pair-wise combinations.

## PARTIAL OVERLAP

In addition to the notions of pair-wise overlap and total overlap, as developed in the previous sections, one can also define the concept of partial overlap. Partial overlap is in some sense a weaker quantification of the stronger notion of total overlap. Partial overlap is a measure of any form of overlap, hence the name partial overlap, i.e. it includes total and pair-wise overlap. The concept is best illustrated by modifying Figure 4 (total overlap diagram) in order to highlight the additional areas where overlap occurs, as shown in Figure 5.

The partial overlap formula given in equation 7 is simply the ratio of the total market capitalisation of all overlapping elements within all of the funds over the combined market capitalisation of all funds. One notes that instead of calculating the total market capitalisation of all overlapping elements, it is easier to calculate the market capitalisation of all non-overlapping elements and then subtract this from the combined market capitalisation of all the funds, as has been done in the equation below. Suppose we have  $N$  funds  $A_1, A_2, \dots, A_N$ . The partial overlap between the  $N$  funds is given as:

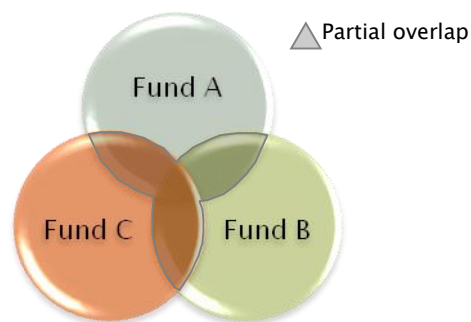


Figure 5: Partial overlap

$$\text{Partial Overlap}_{A_1, A_2, \dots, A_N} = \frac{\sum_{i=1}^N A_i - \bigcup_{i=1}^N (A_i - \bigcup_{i \neq j} A_j)}{\sum_{i=1}^N A_i} \quad 7$$

## LONG-SHORT FUNDS AND FUND NORMALISATION

The exposition of overlap analysis, as presented in this article, was applied to long only funds. However, the concept is easily extended to long-short funds. When calculating the overlap for long-short funds, using the formulas presented above, all we need do is take the absolute (positive) market value of short holdings for any given fund and match them with the short holdings of the same type in other funds for which the overlap analysis is being carried out. In this way we can now calculate all of the relevant quantities needed to measure overlap, such as the market capitalisation of common shares and shares not in common as well as the market capitalisation of individual funds.

At this point it is worth noting that the overlap technique matches the quantity of like holdings within funds, and on the basis of this a figure for the overlap is calculated. However, one could choose to do a normalisation which compares the percentages of the different holdings that are the same within funds. Normalising has the advantage of being able to compare funds that have disproportionate market capitalisations but have similar holdings. Calculating overlap based on holdings, as opposed to the normalised percentage holdings, we will find that if two funds have disproportionate market capitalisations their overlap will be small. However, if the funds have similar holdings with similar weightings their respective percentage returns will be similar. Therefore, depending on the application of the overlap analysis, one may choose to normalise funds before calculating the overlap.



think Notes.

Overlap A,B,C =  $\frac{58,461.00 + 3 \leftarrow 7,961.50}{7,961.50} = 29,01\%$

### VISUALISING PAIR-WISE OVERLAP FOR MULTIPLE FUNDS

Heat maps, which are typically used to visualise data between variables, can be used to show pair-wise overlap values between different pairs of funds. This is also a very convenient method of summarising the pair-wise overlap values when a large number of funds have to be considered. Visualising overlap like this might be particularly useful to investment consultants. Using heat maps gives consultants an easy way to investigate and demonstrate to clients the degree of commonality of the different funds that they are invested in. Heat maps could also be effectively used by Fund-of-Fund managers where decisions regarding the suitability of different funds in terms of diversification have to be made.

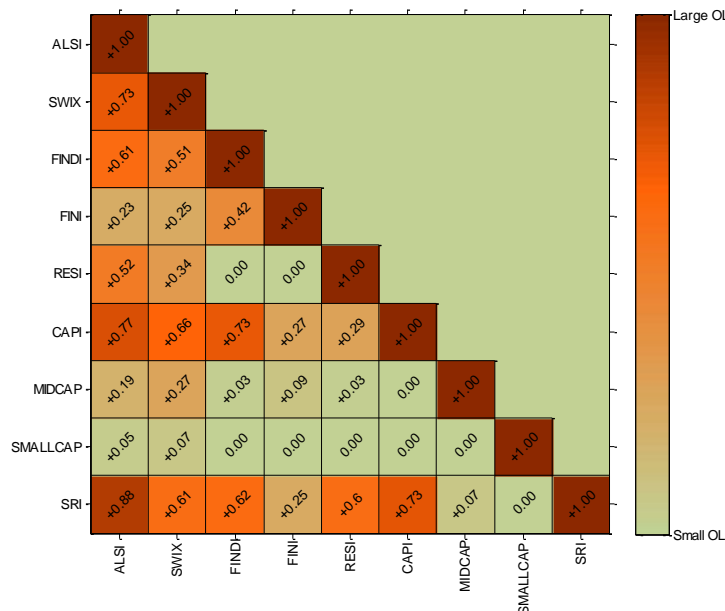


Figure 6: Index overlap heat-map

To demonstrate the power of using heat-maps as a visualisation tool, we have quantified the pair-wise overlap of some of the major indices on the Johannesburg Stock Exchange as at 2008/09/11, see figure 6. Referring to this heat map, we can at a glance make a number of important observations. As is well known, resource companies make up a large part of the South African economy. This is supported by the pair-wise overlap value between the RESI and ALSI indices of 52%. Also widely recognised is the fact that the economy is top heavy, i.e. the market capitalisations of small cap and mid cap stocks do not form a significant part of the economy. This is again supported by the pair-wise overlap values of 5% and 19% respectively for the ALSI and the SMALLCAP indices and the ALSI and MIDCAP indices.

### A RECAP

A method for measuring overlap between funds on a holdings level was introduced. The method is intuitive, therefore it is easily understandable and easily implementable. We have shown how to measure pair-wise, total and partial overlap for these funds. The method can be applied to long only funds as well as long-short funds. In addition to using the number of holdings within a fund, one can do a normalisation by using percentage holdings, which then allows one to compare funds with disproportionate market capitalisations.

Pair-wise overlap analysis of multiple funds was presented visually using heat maps. This has the advantage of condensing numerous pair-wise overlap values into a single chart. The usefulness of these proposed heat maps were demonstrated by analysing some of the major indices on the JSE. The various applications of overlap analysis were pointed out. These include, but are not limited to, transitions, risk analysis and investment consulting.

### Contact details

**Cape Town** Tel: +27 21 673 6999 Fax: +27 21 673 6998; 5th Floor, Montclare Place, cnr of Campground and Main Road, Claremont, 7735; P O Box 23983, Claremont, 7735, Cape Town, South Africa  
**Johannesburg** Tel: +27 11 214 9800 Fax: +27 11 214 9801; Ground Floor, 23 Melrose Boulevard, Melrose Arch, 2076; Postnet Suite 116, Private Bag X1, Melrose Arch, 2076, Johannesburg, South Africa  
[www.riscura.com](http://www.riscura.com)