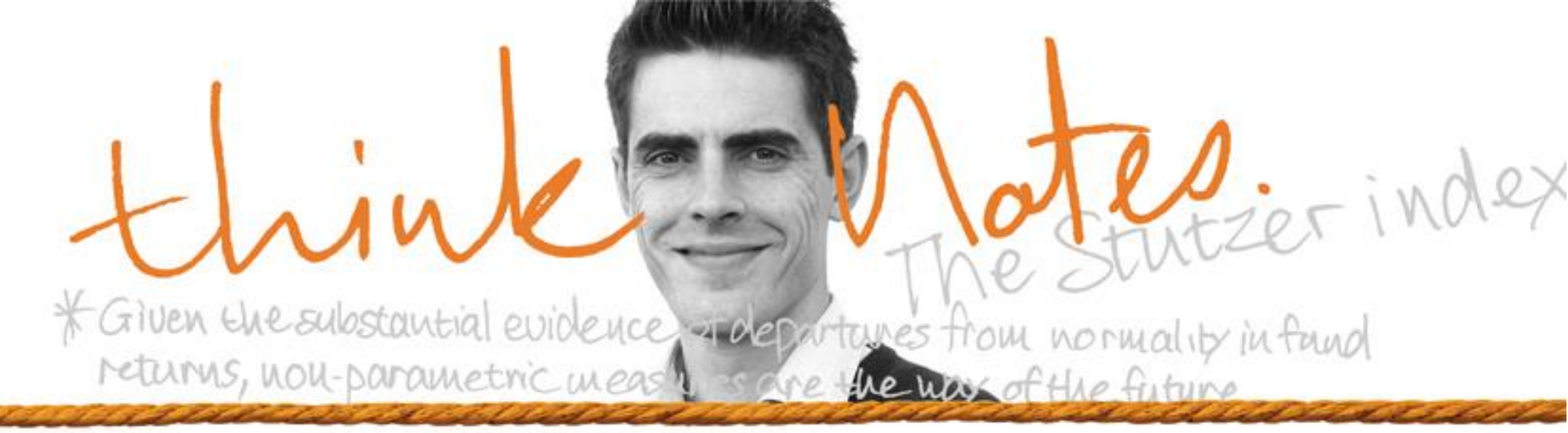

INTRODUCING THE STUTZER INDEX *by Conrad Visagie*

INTRODUCTION

There are various problems associated with traditional risk adjusted performance measures. Most notably, it is usually assumed that the return distributions involved are normally distributed. We know that this assumption sometimes fails miserably, especially if one considers hedge funds where the possible downside is usually limited in some way. Further, risk is associated with the standard deviation of the return distribution even though this definition of risk is clearly insufficient for the typical investor.

A lot of work has been done to construct performance measures which eliminate these inefficiencies, but, with the exception of a handful,[1] this usually leads to a measure that is not as intuitive as the more traditional measures. It is due to this reason that most practitioners default back to the more traditional performance measures, particularly the Sharpe or Information ratio.

The Stutzer index, which will be considered in detail in the next section, solves the abovementioned problems; the notion of risk is based on an intuitive investor behavioural hypothesis and no assumptions regarding normality are made. A couple of other desirable features of this measure will also be discussed. We will also use the Stutzer index to rank a set of funds and see how the ranking obtained differs from a ranking constructed by using the Information Ratio.

THE STUTZER INDEX

Here we introduce the main ideas behind the Stutzer index without going into all of the mathematical technicalities. For a more detailed discussion and derivation of all the results, see [2] and [3]. In the literature the Stutzer index is also referred to as the Portfolio Performance Index or PPI for short.

Before we can start with describing any risk adjusted performance measure, we must define what exactly investors consider as being the risk associated with investing in a particular fund. The Stutzer index is based on the following behavioural hypothesis:

> Investors associate risk with the failure to achieve a certain target return i.e. a fund underperforming its designated benchmark.

[1] The Omega Ratio deserves to be mentioned here. The interested reader can consult [1] for an introductory discussion.

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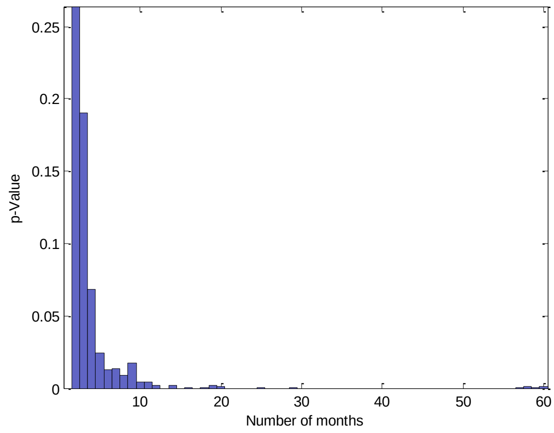
It is possible to show that when a fund is expected to earn a higher average return than the designated benchmark, the probability that the fund will underperform the benchmark approaches zero as we increase the time period under consideration. We set up a hypothesis test with null-hypotheses stating that the fund will underperform the designated benchmark. Figure 1 shows the p-values as we increase the number of months under consideration. The p-value basically states how much evidence we have against the null hypotheses. The lower the p-value, the more evidence we have against the hypothesis that the fund underperforms the benchmark. It is clear from the graph that as we increase the number of months under consideration the p-values decrease.

Figure 1: p-Values versus time period



The Stutzer index ranks funds according to how fast the probability of underperformance decays; the faster the decay rate is, the higher the particular fund will be placed in a ranking table.

If we denote the average excess return of a fund above some benchmark by $\overline{R}(T)$, where T is the length of the sample period under consideration, then

$$\Pr\left(\overline{R_p}(T) \leq 0\right) \approx \frac{c}{\sqrt{T}} \qquad (1)$$

where c is some constant, and I is the decay rate (see [3] for a discussion). We can see that the probability of the fund underperforming the benchmark for large T, decays to zero exponentially. The Stutzer index is now defined as being this decay rate, I, and is given by

$$I = \max_{\theta<0}\left(-\ln\frac{1}{T}\sum_{t=1}^{T} e^{\theta R_t}\right), \qquad (2)$$

where θ is an optimization parameter and $\overline{R}(T)$ is the excess return of the fund above the benchmark at time t. Without getting too technical, it should be noted that equation (2) is a sample estimate of the true Stutzer index, i.e. the I in equation (1), and assumes that fund returns are independent and identically distributed. For a more complete discussion of the above, the reader is urged to consult [2].

The Stutzer index estimates how fast the probability of underperforming the benchmark decays to zero. It is clear that the fund with the largest Stutzer index is preferred. The Stutzer index possesses a remarkable property: if returns are normally distributed, the ranking produced by using the Stutzer index is identical to a ranking produced by using the Sharpe ratio or the Information ratio[2] , depending on if we use the risk-free rate as a benchmark or not. Due to

[2] For a proof of this result, consult [2] and [3].

the popularity of the Sharpe and Information ratios, this is indeed a very desirable feature for a risk adjusted performance measure to have.

Readers might have realized that a problem can potentially arise when we rank funds according to the Stutzer index; the derivation of the index relies on the assumption that the fund outperforms the designated benchmark. This difficulty is easily dealt with: divide the funds under consideration into two groups; those who beat the benchmark and those that do not. Construct a ranking for the group that beats the benchmark according to equation (2). For the group that does not beat the benchmark, rank them according to equation (2) with $\theta > 0$. For this group, a slower decay rate is better and these funds are necessarily ranked lower than funds in the first group.

HIGHLIGHTING POSITIVE SKEWNESS

Another property of the Stutzer measure that was not highlighted in the above discussion is that preference is given to funds that have positively skewed return distributions.[3] We will now demonstrate this property by means of an example.

Consider the three funds A, B and C as well as their corresponding excess return distributions (see figures 2, 3 and 4), were we used an equally weighted average of the three funds' returns to construct a benchmark. All of the return distributions have the same mean and variance $\mu = 0.0051$ and $\sigma = 0.0018$ respectively); they have different skewness ($\gamma_A = -0.73$, $\gamma_B = 0.68$ and, $\gamma_C = 0.00$) and kurtosis.

Figure 2: Return distribution A

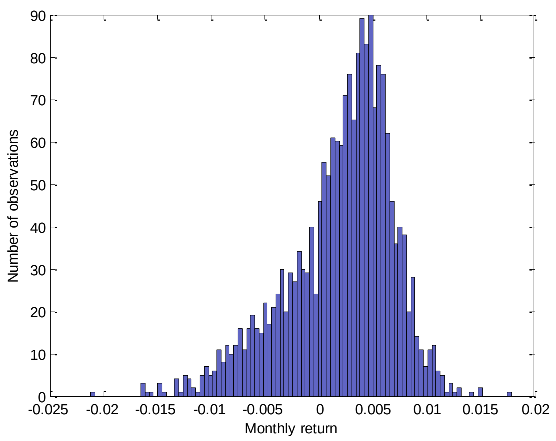
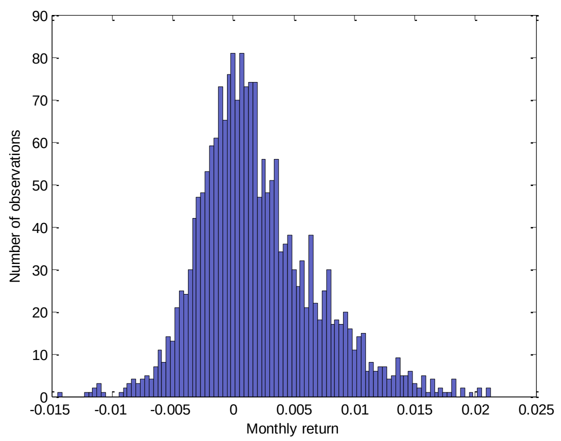
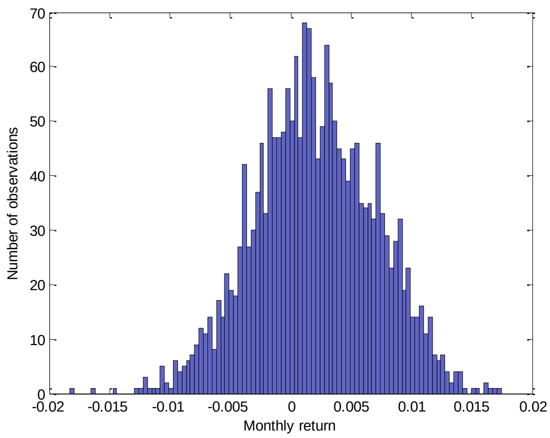


Figure 3: Return distribution B



3 The excess return distributions were simulated as to give the same mean and variance but different skewness. When we calculated the Stutzer index, we only used a subset of 60 observations for each fund.

Figure 4: Return distribution C



Return distribution A is negatively skewed, B is positively skewed and C has zero skewness. If we construct a ranking for these three funds by using the Stutzer index, fund B is ranked first, fund C second and fund A third. This demonstrates that positive skewed returns are favoured by the Stutzer index. The well known Information ratio would have ranked these three funds the same while fund B is clearly preferred due to its positive skewness.

IMPLEMENTATION

As an example we considered a subset of eleven funds from the Global Absolute Return group of funds.[4] We considered monthly returns for the period October 2003 until September 2008. The STEFI Call Deposit Index was used as a risk-free benchmark. Our modus operandi was as follows: compute the expected average excess return above the benchmark. If this value is positive we use the normal Stutzer index. On the other hand, if the value is negative, we use the "modified" Stutzer Index. Our findings are summarized in table 1.

Table 1: Summary of statistics and rankings

Fund	Outperformance rank	Information Ratio rank	Stutzer Index rank	Skewness	Kurtosis	Normal
Fund A	3	2	2	-0.42	-0.08	Yes
Fund B	1	4	3	0.10	-0.42	Yes
Fund C	4	8	8	0.40	-0.11	Yes
Fund D	2	1	1	-0.20	-0.44	Yes
Fund E	8	6	6	0.12	-0.78	Yes
Fund F	11	11	11	-0.11	-0.93	Yes
Fund G	10	10	10	-0.22	-0.69	Yes
Fund H	5	7	7	-0.27	-0.25	Yes
Fund I	6	3	5	-0.47	0.09	Yes
Fund J	9	9	9	-0.38	0.11	Yes
Fund K	7	5	4	-0.08	-0.52	Yes

[4] This is a category in the RisCView survey published by RisCura each month. The latest survey can be found at http://www.riscura.com/surveys_riscview.htm.

We can see that for most of the funds the Information Ratio ranking is the same as the Stutzer Index ranking. This result is plausible since the return distributions of the funds under consideration do not significantly differ from a normal distribution (we used the Jaque-Bera test with a significance level of 5%). We also tested the above returns series for serial autocorrelation out to six lags; we found no statistical evidence of autocorrelation at the 5% significance level. This result validates our use of the sample estimate, equation (2).

RECAP

The Sharpe and Information ratio is widely employed in practice, while they are predicated on the assumption that the fund returns are normally distributed. Even though their use was validated for the sample of funds we considered, this may not always be the case. Furthermore, these metrics fail to capture an investor's downside risk aversion. The Stutzer Index introduced in this article solves the abovementioned problems. The measure is non-parametric and captures an investor's preference for positively skewed returns. Given the substantial evidence of departures from normality in fund returns, non-parametric measures are the way of the future.

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